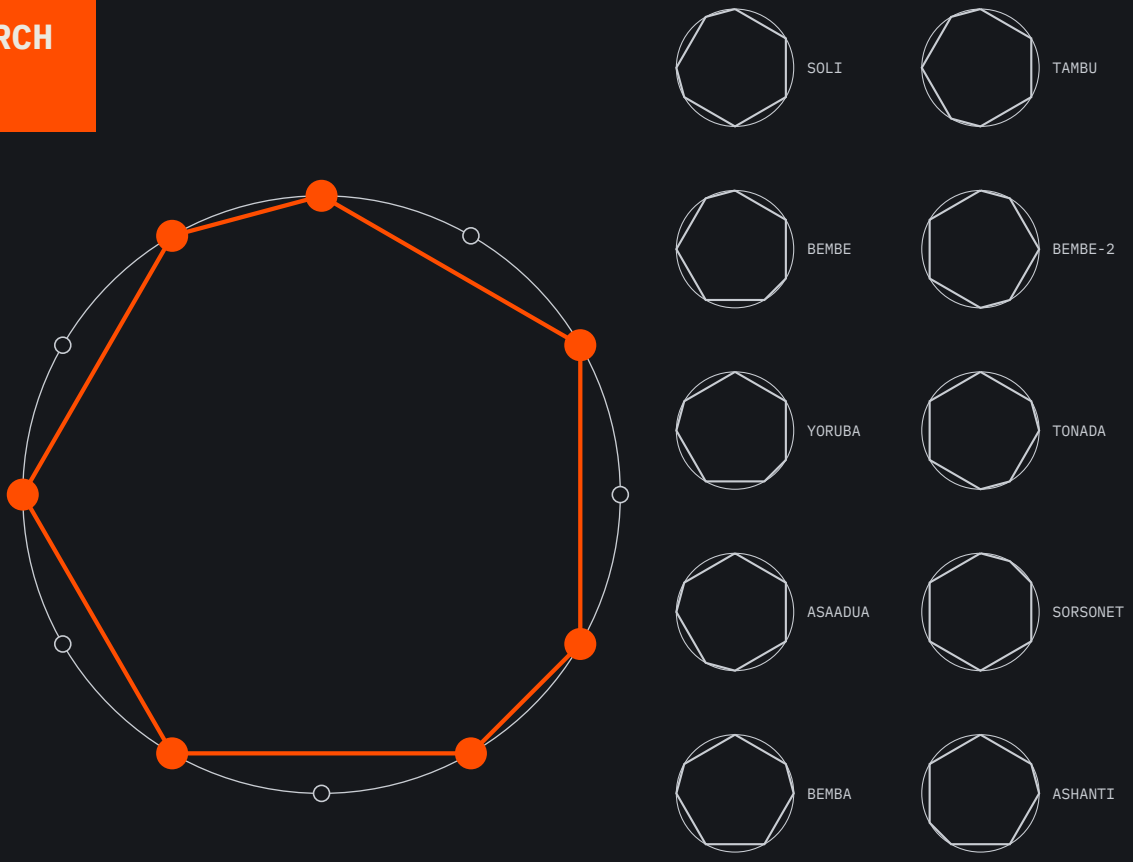
RESEARCH NOTE



THE TEN SEVEN-STROKE BELLS ON TWELVE PULSES (TOUSSAINT 2003); THE STANDARD PATTERN LARGE

PERCUSSION

HOW TRADITIONS PLAY TIME

A survey for sequencers and drum models

> *bells and timelines from West Africa to Cuba and Brazil · the drum as speech: dùndún, manguaré, tabla, solkattu, dum and tek · interlocking: kotekan and the Aka · gamelan's gong cycles · ma, the space in Noh and taiko · Euclid, evenness and oddity · 64 patterns as data, every structural claim asserted · what RUDE and SQRL-1 took, and what they could take next*

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ABSTRACT

Many percussion traditions keep time with two layers: a fixed reference (a bell timeline, a gong cycle, a tal, a darb) and free playing measured against it. This note surveys eight ways of doing that, chosen for what they do with time: the West African bell and its diaspora, the drum that speaks (dùndún, Bora slit drums, tabla bols, solkattu, Arabic dum and tek), the interlocking of Balinese kotekan and the Aka, the nested gong cycles of Javanese gamelan, and the stretched beats and calls of Noh. The mathematics follows: Toussaint's Euclidean rhythms, maximal evenness, Arom's rhythmic oddity and aksak typology, off-beatness, swap distance and complements. The last section sorts what RUDE and SQRL-1 already took and what they could take next, as sequencer modes, generators and physical drum models.

CONVENTIONS

Patterns are written on a pulse grid, x an onset and . a silent pulse, one character per pulse. Onset positions are counted from 0. Beats inside a named cycle (tal, gongan) are counted from 1, as the tradition counts them. "Necklace" means a pattern up to rotation. "Intervals" are the gaps between successive onsets, wrapping round.

1 / HOW TRADITIONS COUNT TIME

rhythm-systems.md has the counting systems in full (tala, the 35 Carnatic talas, aksak, usul, iqa', the Aka). Four distinctions from it matter for a drum part.

1.1 THE REFERENCE AND THE FREE PART

West Africa. Agawu (2006) calls the seven-stroke, twelve-pulse bell figure the "standard pattern" and argues it has to be read both structurally and culturally; it is the fixed reference that drums and dancers are measured against. K. Agawu, Structural analysis or cultural analysis? Competing perspectives on the "standard pattern" of West African rhythm, JAMS 59(1), 2006, doi:10.1525/jams.2006.59.1.1. In Locke's account of Ewe agbadza, the gankogui (bell), axatse (rattle) and kagan (small drum) keep an unvarying background while the sogo and kidi drums carry drum language. D. Locke, Agbadza: The Critical Edition, Tufts University, 2011, doi:10.60965/CT6Z-9W70. The role split is from a search excerpt of Locke's text, not checked against the edition.

For a sequencer: two layers, a locked reference lane and lanes that vary against it, is the architecture. The reference never mutates.

1.2 THE COUNT AS A GESTURE

Hindustani tal. The cycle is kept with claps (tali) and a wave (khali). Teental is 16 beats in four vibhags of 4; claps on 1, 5 and 13, the wave on 9. https://en.wikipedia.org/wiki/Teental . The tabla's theka fills the count with strokes (section 3.3).

Carnatic tala and gati. The tala fixes the cycle (Adi, 8 beats); the gati fixes how many syllables fill a beat (3, 4, 5, 7 or 9). Section 3.4.

1.3 THREE LETTERS, NOT TWO

Arabic darb. Frishkopf's chart builds each cycle from three letters: DUM (low, strong), TEK (high, weaker) and ISS (least accented, left blank, the place for fills). M. Frishkopf, Some Arab rhythmic cycles (durub, awzan, iqa'at), chart, 2009, https://sites.ualberta.ca/~michaelf/Theory/Durub%20chart%20rev%202009.pdf .

For a sequencer: a step is not on/off but one of three states, and the third (iss) is where the player improvises. A darb lane could hold dum and tek fixed and generate only inside iss.

1.4 AKSAK IS ABOUT THE SUM

Arom (2004) classifies aksak (fast cycles of 2s and 3s) by the sum of its pulses: "authentic" if the sum is prime (5 = 2+3, 7 = 2+2+3, 11, 13), "quasi-aksak" if odd but not prime (9 = 2+2+2+3), "pseudo-aksak" if even (8 = 3+3+2, 10 = 2+2+3+3, 12 = 2+2+2+3+3). S. Arom, L'aksak: principes et typologie, Cahiers de musiques traditionnelles 17, 2004, 11-48, https://journals.openedition.org/ethnomusicologie/413 . The word and the first study are Brăiloiu's: C. Brăiloiu, Le rythme aksak, Revue de musicologie 33, 1951, 71-108.

A prime sum cannot be split into equal beats at any level; a sum divisible by 3 can be heard as three equal beats; an even sum as two. The 3+3+2 pseudo-aksak is the tresillo necklace (asserted). For a generator: pick the sum's class first, then distribute 2s and 3s.

2 / TIMELINES AND BELLS

2.1 THE STANDARD PATTERN (EWE, YORUBA; CUBA, HAITI, BRAZIL)

Instrument. An iron bell: the gankogui in Ewe agbadza and agbekor; the guataca, a hoe blade, in Cuban batá and bembé. Pattern (Toussaint's "Bembé"): x.x.xx.x.x.x, intervals 2 2 1 2 2 2 1, onsets 0 2 4 5 7 9 11. Toussaint lists it among the Ewe and Yoruba, in agbekor and agbadza, the Mongo zebola, Guinean tiriba, and in Cuba, Haiti (ibo) and Brazil. G. Toussaint, Classification and phylogenetic analysis of African ternary rhythm timelines, Bridges 2003, 25-36, https://archive.bridgesmathart.org/2003/bridges2003-25.html .

Toussaint shows three properties that single it out among the ten seven-stroke bells: it is maximally even, it is the only one with off-beatness 3 (onsets on three of the four 12-pulse positions, 1 5 7 11, that no 2-, 3-, 4- or 6-gon touches), and it is the diatonic scale in time (section 7). All three are asserted in the CSV script.

For a sequencer: the bell is the clock that everything else is heard against. A lane flag "reference" (never mutated, never rotated by a blend) is the useful feature. For a model: two-tone iron bells and a struck hoe blade are bar and plate modes, strongly inharmonic, short. Whether the gankogui in agbadza plays both bells in the standard pattern is unchecked.

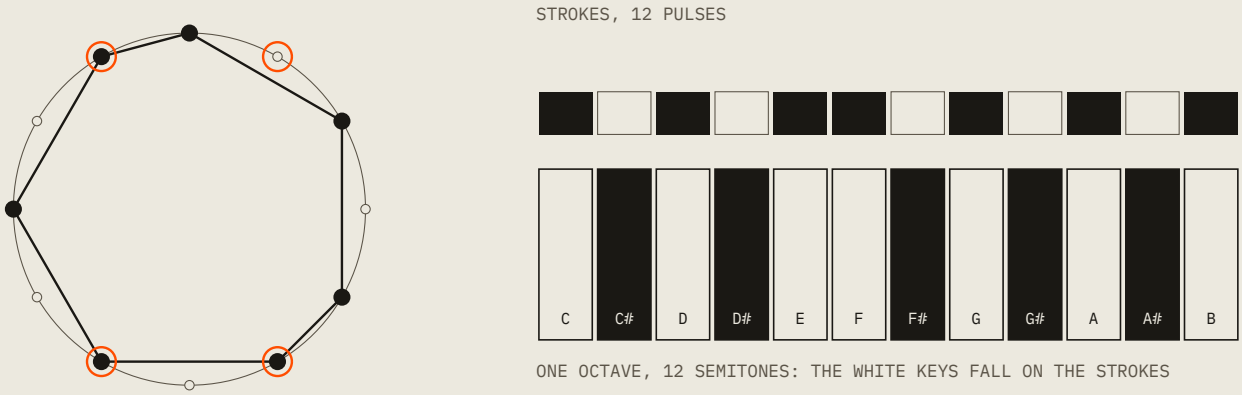


fig. 1: the standard pattern x.x.xx.x.x.x. Left: as a necklace, pulse 0 at the top, clockwise; the accent rings mark pulses 1, 5, 7 and 11, which no even pulse stream touches; three strokes fall there (off-beatness 3). Right: the same pattern over one octave, the diatonic scale in time (Pressing 1983).

2.2 THE TEN BELLS AND THE WHEEL

Toussaint (2003) finds ten seven-stroke 12-pulse bells in use, every one with intervals two 1s and five 2s: Soli, Tambú, Bembé, Bembé-2, Yoruba, Tonada, Asaadua, Sorsonet, Bemba, Ashanti (CSV, section timelines). Played from each of its seven onsets in turn (the "Bembé wheel", a teaching exercise he credits to Gary Harding), the standard pattern gives six of them; the third rotation, xx.x.x.xx.x., he could not find in any tradition.

The ten fall into three necklaces, by where the two short intervals sit: Sorsonet alone (the 1s adjacent); Tonada, Asaadua and Soli (the 1s one 2 apart); the six of the Bembé wheel (the 1s as far apart as they can be). Toussaint names the Tonada wheel's fifth rotation as the Persian al-ramal; the script shows it is the Soli. The Cuban bembé can carry two bells at once, the standard pattern on the guataca and Bembé-2 (xx.x.xx.x.x.) on a low bell, which is the same necklace from another onset.

For a sequencer: rotation by onset, not by step. SQRL-1's ROT and RUDE's Rotate move by steps; most step rotations of a timeline are not traditional, its onset rotations mostly are.

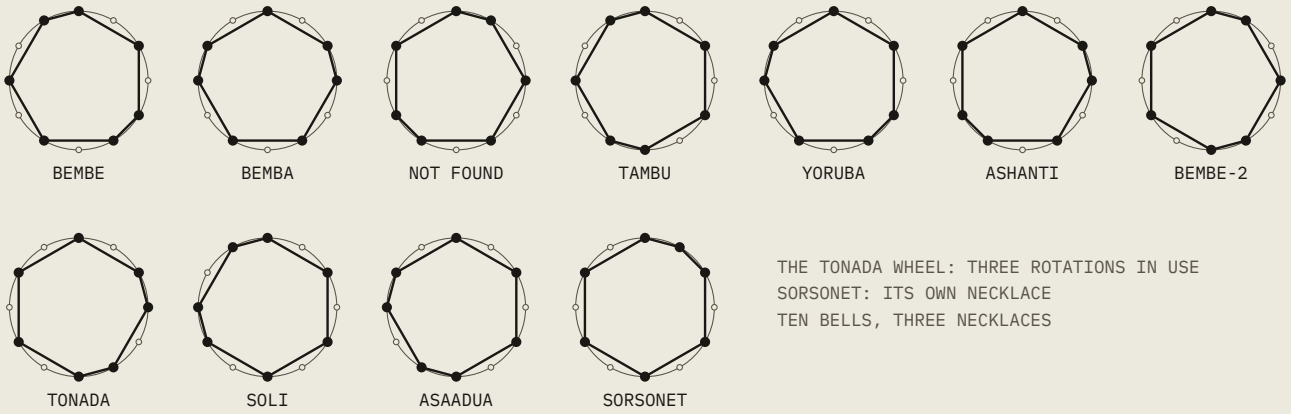


fig. 2: the Bembé wheel: the standard pattern started on each of its seven onsets. Six rotations are bells in use; the third is not found in any tradition (Toussaint 2003). Below, the other two necklaces of the ten seven-stroke bells.

2.3 THE 4/4 FAMILY: CLAVE AND ITS RELATIVES

Toussaint (2002) compares six five-stroke 16-pulse timelines (his names):

name	pattern	intervals
Shiko	x...x.x...x.x...	4 2 4 2 4
Son clave	x..x..x...x.x...	3 3 4 2 4
Soukous	x..x..x...xx....	3 3 4 1 5
Rumba clave	x..x...x..x.x...	3 4 3 2 4
Bossa-nova	x..x..x...x..x..	3 3 4 3 3
Gahu	x..x..x...x...x.	3 3 4 4 2

G. Toussaint, A mathematical analysis of African, Brazilian, and Cuban clave rhythms, Bridges 2002, 157-168, https://archive.bridgesmathart.org/2002/bridges2002-157.html . The figure is a scan; the notation above is reconstructed and held to every property the text states: Shiko and Bossa-nova are palindromes, Son a palindrome about pulse 3; Shiko, Soukous and Gahu (which the text says evolved in Africa) have a right angle in their polygon, that is, two onsets opposite; Son, Rumba and Bossa-nova (America) do not; the isosceles counts are 1, 1, 1, 2, 3 and 0 for Rumba; Bossa-nova is maximally even. Son has Arom's rhythmic oddity. Its first bar is the tresillo: Son is "tresillo then two". Toussaint 2005 (below) gives the son string directly.

For a sequencer: the clave is a two-bar reference whose halves are unequal (3-2). A lane that can be flipped between 3-2 and 2-3 (a rotation by half the cycle) is the musician's control. Son is not Euclidean, so FILL and ROT alone cannot make it.

2 / TIMELINES AND BELLS

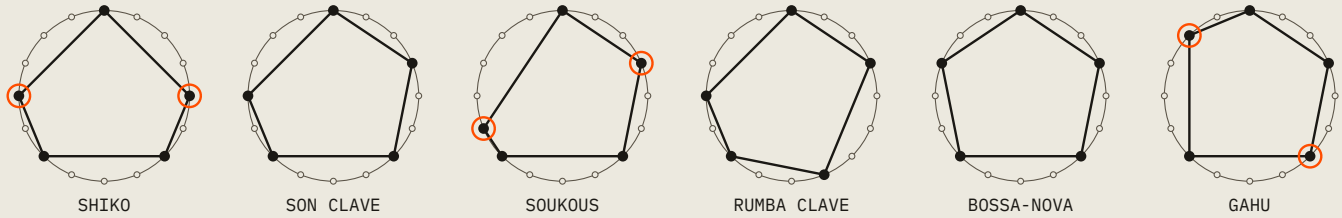


fig. 3: Toussaint's six five-stroke timelines on 16 pulses (2002). Accent rings mark two onsets directly opposite, a right angle in the polygon: Shiko, Soukous and Gahu have one; son, rumba and bossa-nova do not. Son's first half is the tresillo.

2.4 TRESILLO AND CINQUILLO

x..x..x. (E(3,8)) and x.xx.xx. (E(5,8)). Toussaint (2005) finds the tresillo on the atoke bell of the Ewe sohu, as the first bar of son clave, and in 1950s rock and roll bass lines; the cinquillo in Cuba, West Africa and jazz. G. Toussaint, The Euclidean algorithm generates traditional musical rhythms, Bridges 2005, 47-56, https://archive.bridgesmathart.org/2005/bridges2005-47.html .

2.5 SAMBA AND BOSSA (BRAZIL)

Toussaint (2005): the samba rhythm x.x..x.x.x..x.x. is E(7,16) started on its last onset; a samba cowbell part is E(9,16) started on its fourth onset (x.x.xx.x.x.x.xx.); the bossa-nova is E(5,16) started on its third onset. In a batucada the first surdo accents beats 2 and 4, a second surdo, tuned higher, beats 1 and 3, a third syncopates between; the repinique leads. https://en.wikipedia.org/wiki/Batucada . The cuíca, a friction drum, rises in pitch the harder the finger presses the head (same source).

For a sequencer: the surdo pair is call and response at the beat level, two lanes of one instrument at two tunings. For a model: the cuíca is a friction-driven membrane with continuous pitch, a stick-slip exciter like a bowed string's.

3 / THE DRUM AS SPEECH

3.1 DÙNDÚN (YORÙBÁ, NIGERIA)

Instrument. A double-headed drum hung from the shoulder; tension strings (ọsán) run between the two heads; the left hand, wrist and hip squeeze them, a curved stick (ọ̀pá, kọ̀ngọ̀) strikes. Minimal pressure gives the lowest pitch, maximum pressure the high tone. Durojaye et al. (2021) recorded three speakers and three drummers in four modes, from dance rhythm to direct speech imitation. In direct imitation, drum and speech timing correlate at r = 0.72 (inter-onset intervals); in the performative mode, 0.37. C. Durojaye, K. Knowles, K. Patten, M. Garcia, M. McBeath, When music speaks: an acoustic study of the speech surrogacy of the Nigerian dùndún talking drum, Frontiers in Communication 6, 2021, doi:10.3389/fcomm.2021.652690 . Akinbo (2019) measures three distinct drum f0 levels for Yorùbá's low, mid and high tones on the gángan. S. Akinbo, Representation of Yorùbá tones by a talking drum: an acoustic analysis, 2019, lingbuzz/004884 (under revision when found).

For a sequencer: each step carries a pitch contour, not a note: L, M, H and the glides between them. Text with tone marks could drive it. For a model: a membrane whose tension is a continuous control during the decay. Pitch bend is the instrument, not an effect.

3.2 MANGUARÉ (BORA, AMAZONIA)

Instrument. A pair of wooden slit drums, each with two pitches, four in all. Seifart et al. (2018) find that the talking mode carries words mainly by rhythm: four rhythmic units in the pauses between beats, matching vowel-to-vowel intervals; the two drummed tones carry few lexical contrasts. F. Seifart, J. Meyer, S. Grawunder, L. Dentel, Reducing language to rhythm: Amazonian Bora drummed language exploits speech rhythm for long-distance communication, Royal Society Open Science 5, 2018, 170354, doi:10.1098/rsos.170354 .

For a sequencer: the opposite of the dùndún. Here the gap, not the pitch, carries meaning: four quantised pause lengths. A speech-to-rhythm mode could take the gaps from a text's syllable structure. For a model: a slit drum is a wooden plate-and-cavity resonator, two lips of different thickness for two pitches.

3.3 TABLA BOLS AND THEKA (HINDUSTANI)

Instrument. Two drums: the dayan (right, tuned, a loaded membrane with the black syahi patch) and the bayan (left, bass, its pitch bent by the heel of the hand; this description of the pair is general knowledge, unchecked here). Bols are the spoken names of strokes. The teental theka is dha dhin dhin dha | dha dhin dhin dha | dha tin tin ta | ta dhin dhin dha; https://en.wikipedia.org/wiki/Teental ; https://digitabla.com/reference/compositional-forms/theka/ . Dha and dhin include the bayan's open bass; tin and ta do not (unchecked against a stroke reference). The khali vibhag is heard as the bass dropping out, not as a rest. Digitabla notes that the theka's bass-less bols fall on matras 10 to 13, one beat later than the khali vibhag of 9 to 12 (from a search excerpt of the theka page).

Raman explained the tabla's and mridangam's unusually harmonic tone: the loaded membrane's modes fall near a harmonic series. C. V. Raman, The Indian musical drums, Proc. Indian Acad. Sci. A 1, 1934, 179-188, doi:10.1007/BF03035705 ; earlier, C. V. Raman and S. Kumar, Musical drums with harmonic overtones, Nature 104, 1920, 500 (doi not checked). A later paper summarises Raman's dayan as a fundamental and four harmonic overtones (arXiv:1601.02489), and D. Courtney (1999) calls the model an approximation (both read only in search excerpts).

For a sequencer: a bol is a stroke on two voices at once; khali is a voice muted for a section. A theka lane could carry bols and render them to two drums. For a model: the dayan is the case where a membrane is engineered to be harmonic; a modal model with mode ratios near 1:2:3:4:5 plus a damped-stroke family (ta, tin, na) is the target.

3 / THE DRUM AS SPEECH

3.4 SOLKATTU AND KONNAKOL (CARNATIC)

Instrument. The voice, recited against the tala clapped by hand; the mridangam plays the same phrases. Group syllables by gati: tha ki ta (3), tha ka dhi mi (4), tha dhi gi na thom (5), tha ka di mi tha ki ta (7), tha ka dhi mi ta dhi gi na thom (9). https://en.wikipedia.org/wiki/Konnakol , after D. P. Nelson, Solkattu Manual, Wesleyan, 2008. Cadences called mora and korvai repeat a phrase (usually three times) so that it ends on the first beat of the cycle (sam). The definitions of mora and korvai are unchecked; Nelson's Konnakkol Manual (Wesleyan, 2019) covers them.

For a sequencer: a phrase of length p repeated three times with gaps g, landing on sam, solves 3p + 2g = (beats left in the cycle); a "land on one" generator is exact arithmetic.

3.5 DUM AND TEK (ARABIC DARB)

Instrument. Darbuka (goblet drum), riqq, Arab tabla: the dum is the low centre stroke, the tek the high edge. Frishkopf's positions, on an eighth-note grid (CSV, section drum-as-speech):

darb	strokes
Maqsum	D T . T D . T .
Sa'idi	D T . D D . T .
Masmudi saghir (baladi)	D D . T D . T .
Malfuf	D . . T . . T .
Ayyub	D . . T D . T .
Wahda kabira	D . . (T) . . (T) .
Dawr hindi (3.5 beats)	D T T D . T .
Aqsaq (9)	D . T . D . T . T
Sama'i thaqil (10)	D . . T . D D T . .

Maqsum, sa'idi and baladi share one onset set and differ only in which strokes are dum: the colour of a stroke is part of the rhythm's identity. That onset set is the cinquillo necklace; malfuf sits on the tresillo's positions (both asserted). Toussaint (2005) lists E(5,8) as the Egyptian malfuf, where Frishkopf's framework has three strokes: a realisation with fills against a bare frame, unresolved.

For a sequencer: two lanes are not enough to name a darb; a stroke-colour lane over a shared onset lane is. Swapping colours (maqsum to sa'idi) is a one-parameter variation.

3 / THE DRUM AS SPEECH

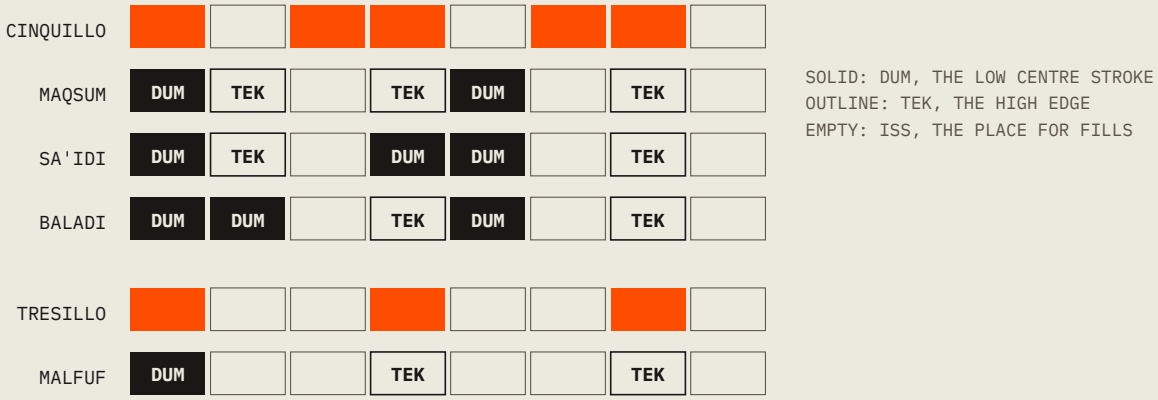


fig. 4: three darb share one onset set, the cinquillo's necklace started on its second onset, and differ only in which strokes are dum; malfuf sits on the tresillo's positions (Frishkopf 2009). Eight pulses, one eighth note each.

4 / INTERLOCKING AND HOCKET

4.1 KOTEKAN (BALI)

Instrument. Gangsa (metallophones) and reyong (gong chimes) in gamelan gong kebyar, in pairs: the polos part sits mostly on the beat, the sangsih off it. In kotekan telu both parts use three pitches and share the middle one. Wikipedia's example, pitches 1 2 3 on 16 pulses:

https://en.wikipedia.org/wiki/Kotekan (after C. McPhee, Music in Bali, 1966; M. Tenzer, Balinese Music, 1998; H. Spiller, Gamelan, 2004). Together the two fill all 16 pulses; they coincide on pulses 1 4 7 10 13, always on the shared pitch (asserted). In nyog cag the parts simply alternate, every other note; there the sangsih is the exact complement of the polos. The composite runs four or eight times faster than the core melody (same source).

For a sequencer: "split one stream into two players" is the operation: given a composite line, assign each note to polos or sangsih by rule (alternate; or by pitch, sharing the middle). RUDE's Hocket blend goes the other way, merging two parts. For a model: each pair is tuned apart to beat (ombak, voices-and-phantoms.md); a paired-metallophone voice is one model, two tunings.

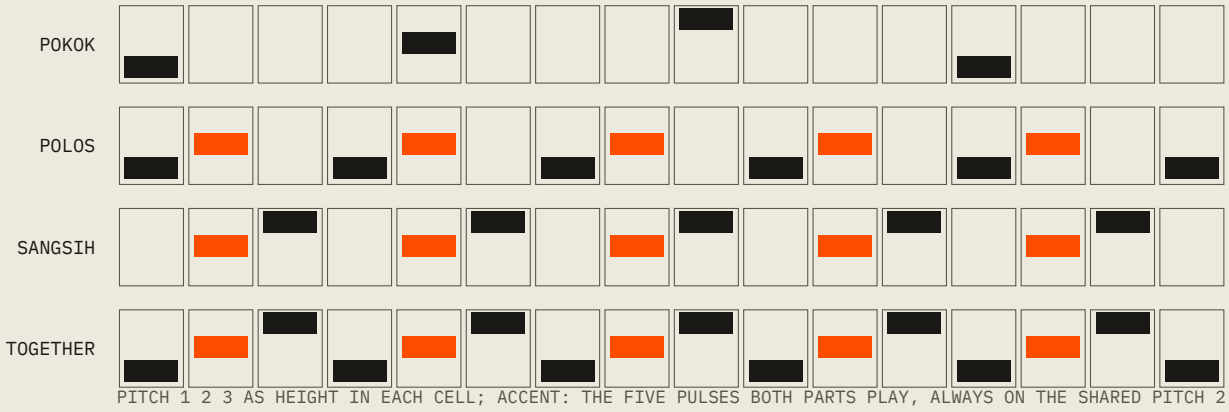


fig. 5: kotekan telu on 16 pulses: the polos mostly on the beat, the sangsih off it, three pitches each with the middle one shared. Together they fill every pulse (after McPhee, Tenzer, Spiller via the Wikipedia summary).

4.2 THE AKA (CENTRAL AFRICA)

The Aka's polyphony interlocks voices and instruments over timelines with Arom's rhythmic oddity (rhythm-systems.md). Toussaint (2005), after Arom (1991), gives two Aka necklaces: E(11,24), "usually started on the seventh onset", and E(13,24), from the upper Sangha, started on the fourth. The script finds that E(11,24) is exactly the Aka pattern Chemillier and Truchet (2003) open with, and that it has the oddity property; E(13,24) does not. S. Arom, African Polyphony and Polyrhythm, Cambridge, 1991; M. Chemillier and C. Truchet, Computation of words satisfying the "rhythmic oddity property" (after Simha Arom's works), Information Processing Letters 86(5), 2003, 255-261, doi:10.1016/S0020-0190(02)00521-5 .

Hindewhu, of the BaBenzélé, alternates sung syllables with a one-pitch papaya-stem whistle, a hocket inside one performer. https://en.wikipedia.org/wiki/Pygmy_music . Arom as the recordist is unchecked.

4.3 TWO BELLS AT ONCE (CUBA)

Section 2.2: the bembé's guataca and low bell play two rotations of one necklace together. Interlocking here is a rotation, not a complement.

4 / INTERLOCKING AND HOCKET

4.4 PHASING: REICH (THE WESTERN BRIDGE)

Reich studied in Ghana in 1970 with the Anlo Ewe master drummer Gideon Alorwoyie; the visit was cut short by malaria. Drumming (1970-71) builds one 12/8 bar by substituting beats for rests, then phases two players a quarter note apart. https://en.wikipedia.org/wiki/Drumming_(Reich) . That Reich first met Alfred Ladzekpo at Columbia, and the five-week length of the visit, are from search excerpts only.

Clapping Music (1972) is xxx.xx.x.xx.. Colannino, Gómez and Toussaint show it is a Yoruba bell timeline with one onset added; the script confirms it is exactly Toussaint's Yoruba bell (x.x.xx.x.xx.) plus pulse 1. J. Colannino, F. Gómez, G. Toussaint, Analysis of emergent beat-class sets in Steve Reich's "Clapping Music" and the Yoruba bell timeline, Perspectives of New Music 47(1), 2009, 111-134, doi:10.1353/pnm.2009.0014 ; earlier version Bridges 2006, https://archive.bridgesmathart.org/2006/bridges2006-49.html . One player shifts by one pulse per section; the pattern has no rotational symmetry, so shifts 1 to 11 all differ from unison and the twelfth returns to it (asserted).

For a sequencer: phasing is interlocking that moves: a second copy of a timeline rotated by one pulse every N bars. RUDE's Duet with Drift already does this.

5 / CYCLES AND COLOTOMY

5.1 JAVANESE GAMELAN: THE GONG CYCLE

Instruments. Gong ageng (largest), kempul (hanging gongs), kenong (large kettles), kethuk and kempyang (small kettles). Each form fixes where each sounds in one gongan (one gong cycle). Symbols: T kethuk, p kempyang, N kenong, P kempul, G gong with kenong, W wela (the kempul's place left silent).

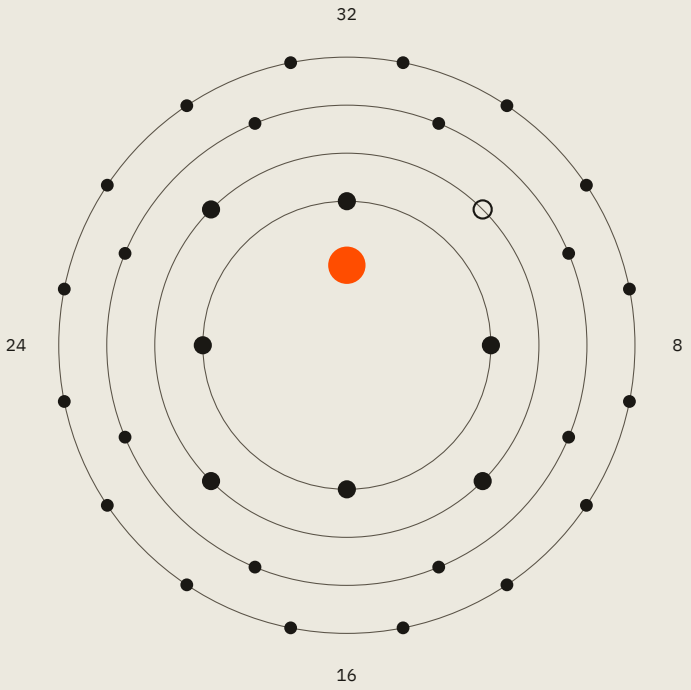
form	beats	pattern
lancaran	16	T W T N T P T N T P T N T P T G
ketawang	16	p T p W p T p N p T p P p T p G
ladrang	32	p T p W p T p N p T p P p T p N p T p P p T p N p T p P p T p G

Lancaran: kethuk on odd beats, kenong 4 8 12, kempul 6 10 14, gong 16, wela 2. https://en.wikipedia.org/wiki/Lancaran . Ladrang: kempyang on odd beats, kethuk 2 6 10 ..., kenong 8 16 24 (and 32 with the gong), kempul 12 20 28, wela 4. https://en.wikipedia.org/wiki/Ladrang (after J. Lindsay, Javanese Gamelan, 1992, 48-49; N. Sorrell, A Guide to the Gamelan, 1990, 68-69). Ketawang: https://en.wikipedia.org/wiki/Colotomy (after W. Malm, 1977, 53). All positions asserted.

The structure is a hierarchy: each instrument subdivides the one above, and the largest gong closes the cycle rather than opening it. The wela is a planned absence: the kempul's first place is left silent (why, in the tradition's terms, is unchecked; one reading is that the cycle's start is not marked like its middles). Becker treats the gong cycle as an image of cyclic time. J. Becker, Hindu-Buddhist time in Javanese gamelan music, in J. T. Fraser (ed.), The Study of Time IV, 1981 (cited via Toussaint 2003; not read).

For a sequencer: a colotomic mode is a set of lanes in power-of-two ratios whose downbeat is the end: the gong sounds on the last step. Irama, the change of density against tempo, is the natural macro control (unchecked: the irama levels are not sourced here). For a model: gongs with a pitched, beating hum (the ombak of section 4.1 and a BYU measurement of a Balinese gong beating near 3 Hz, voices-and-phantoms.md).

5 / CYCLES AND COLOTOMY



OUTER TO INNER: KEMPYANG (ODD BEATS), KETHUK (2 6 10 ...), KEMPUL (12 20 28), KENONG (8 16 24 32), AND THE GONG AGENG IN ACCENT AT 32.

THE HOLLOW RING AT 4 IS THE WELA: THE KEMPUL'S PLACE LEFT SILENT.

BEAT 32 IS AT THE TOP: THE CYCLE CLOSES ON THE GONG, IT DOES NOT OPEN WITH IT.

fig. 6: one gongan of ladrang, 32 beats, as concentric rings (after Lindsay 1992 and Sorrell 1990, via the Wikipedia summary). Each instrument subdivides the one outside it; the largest gong ends the cycle.

5.2 USUL ON KUDÜM (TURKEY) AND LONG CYCLES

The six usuls in RUDE (weights from Holzapfel and Bozkurt 2012) and zencir (120 beats) are in rhythm-systems.md. The kudüm is a pair of small kettledrums; the düm and tek of the usul fall on the lower and higher drum. The left/right allocation is unchecked here. For this paper the point is that an usul, like a gongan, is a cycle long enough that its position, not its pulse, is what the player tracks.

5.3 AKSAK ON THE TAPAN (BALKANS)

Instrument. The tapan (davul): a thick stick (kukuda in Bulgarian, tokmak in Turkish) on one head for the accented, low strokes, a thin switch (prachka, çubuk) on the other for light ones. https://en.wikipedia.org/wiki/Davul (stick names after J. Pappas). In an aksak such as the rachenitsa (2+2+3), the group starts are the natural place for the thick stick. That allocation is an illustration from the stick roles, not a transcription.

For a model: two heads, two beaters, two timbres on one shell: a coupled two-membrane model with a cavity, struck on both sides.

6 / SILENCE AND SPACE (MA)

6.1 NOH: CALLS AND STRETCHED BEATS

Instruments. The hayashi: nōkan (flute), kotsuzumi (shoulder drum), ōtsuzumi (hip drum), taiko (stick drum). The drummers' calls (kakegoe) are part of the rhythm. In strict settings the calls sit between beats, short and accented; in flexible settings they start earlier and can fill the whole gap. Stanford Noh project, https://noh.stanford.edu/music/otsuzumi-kotsuzumi (Fujita, Kapuscinski, Rose 2020). In mitsuji-utai, beats 1, 3 and 5 of the eight-beat frame are condensed. https://noh.stanford.edu/music/voices . The call placement RUDE uses ("ya" before 1 and 5, "ha" before 2 3 6 7) and its sources are in plugins/rude/docs/rudiments.md.

The silence is not empty time: the call is the preparation, the stroke the release, and the length of the call sets the length of the gap. Stanford's intermedia essay frames the call and stroke as a build-up and release (https://noh.stanford.edu/noh-as-intermedia , read in excerpt).

For a sequencer: two things a grid lacks. A per-step length (which beats shrink) and a pre-event (the call) whose length decides when the event lands. The mitsuji row in the CSV marks which beats shrink, not by how much. For a model: Stanford's page gives four kotsuzumi sounds (low or lower, strong and resonant; high, strong or weak, damped) and three ōtsuzumi sounds (strong and resonant, weak and resonant, weak and damped). That the kotsuzumi player changes the tone by squeezing the lacing is unchecked here. A physical model of the drum is the place to test it.

6.2 TAIKO: MA AND KUCHI SHŌGA

Kumi-daiko, the ensemble form, was developed by Daihachi Oguchi in 1951 (Osuwa Daiko). Kuchi shōga, spoken syllables, carry rhythm and timbre: "don" one strike to the centre, "do-ko" two quick strikes, right then left, in the time of one don; syllables vary by region. Ma, the space between strikes, shapes phrases. https://en.wikipedia.org/wiki/Taiko (citing Tusler 2003, Bender 2012, Varian 2013). Ka for a rim strike and tsu for a soft stroke are from a teaching page found in search, not checked against Bender or Varian.

For a sequencer: do-ko is a subdivided slot with fixed hands, which RUDE's grammar can already write (two strokes dividing one slot). Ma has no grid form; it is the per-step length of 6.1 again.

6.3 SILENCE AS A MISSING VOICE

Two cases above where silence is a voice dropping out, not a rest: the tabla's khali (the bayan stops) and the gamelan's wela (the kempul's place left empty). The Arabic iss (1.3) is the inverse: a marked place for optional sound.

For a sequencer: per-section voice mutes on a reference cycle (khali, wela) are cheap and idiomatic; SQRL-1's WAKE and NAP are the same idea per bar, not per section.

7 / THE MATHEMATICS

7.1 EUCLIDEAN RHYTHMS

Bjorklund (2003) needed to spread k pulses as evenly as possible over n timing slots in a neutron-source accelerator; his algorithm has the structure of Euclid's for the greatest common divisor. Toussaint (2005) names the results E(k,n) and finds many timelines among them: E(2,5) khafif-e-ramal (13th-century Persian), E(3,7) a ruchenitza, E(3,8) tresillo, E(4,9) Turkish aksak, E(5,7) nawakhat, E(5,8) cinquillo, E(5,9) agsag-samai, E(5,12) Venda clapping, E(5,16) bossa-nova necklace, E(7,8) Tuareg bendir, E(7,12) a West African bell, E(7,16) samba necklace, E(9,16) Central African and samba cowbell, E(11,24) and E(13,24) Aka. E. Bjorklund, The theory of rep-rate pattern generation in the SNS timing system, SNS-NOTE-CNTRL-99, Los Alamos, 2003 (cited via Toussaint). The script checks every one of these strings is a rotation of E(k,n).

Demaine et al. prove that Euclidean rhythms are the unique rhythms maximising a natural evenness measure, and characterise "deep" rhythms (each distance occurs a different number of times). E. Demaine et al., The distance geometry of music, Computational Geometry 42(5), 2009, 429-454, doi:10.1016/j.comgeo.2008.04.005 ; arXiv:0705.4085 . Gómez-Martín, Taslakian and Toussaint show each Euclidean rhythm is a repeated main pattern plus a tail. F. Gómez-Martín, P. Taslakian, G. Toussaint, Structural properties of Euclidean rhythms, Journal of Mathematics and Music 3(1), 2009 (doi not found).

What Euclid does not give: the starting onset (son, clave direction, the Bembé wheel), stroke colour (maqsum against sa'idi), and the non-Euclidean timelines (son, rumba, shiko, gahu, soukous, Soli, Tonada, Asaadua, Sorsonet). 40 of the CSV's 64 rows are Euclidean necklaces and six more are every-pulse rows; the column sqrl marks which.

7.2 MAXIMAL EVENNESS AND THE DIATONIC ISOMORPHISM

Clough and Douthett define a maximally even set as k points spread as evenly as possible round a circle of n. J. Clough, J. Douthett, Maximally even sets, Journal of Music Theory 35(1/2), 1991, 93-173, doi:10.2307/843811 . Pressing (1983) observed that the standard pattern and the diatonic scale have the same interval pattern, white keys for strokes: C D E F G A B on a 12-step chromatic grid is x.x.xx.x.x.x. J. Pressing, Cognitive isomorphisms between pitch and rhythm in world musics: West Africa, the Balkans and Western tonality, Studies in Music 17, 1983, 38-61 (cited via Toussaint 2003 and Demaine et al.; one listing gives the title as "...in world music: West Africa, the Balkans, Thailand and Western tonality"; not read). Toussaint (2003) shows the Bembé is maximally even under Block and Douthett's chord-length measure (asserted by brute force over all 792 seven-in-twelve patterns).

Complement (derived here). The five silent pulses of the standard pattern, .x.x..x.x.x., are a rotation of E(5,12), the Venda and fume-fume timeline: the black keys are the pentatonic in time as the white keys are the diatonic. An interlocking second part that fills exactly the bell's silences is therefore itself a traditional timeline.

7.3 RHYTHMIC ODDITY

Arom's property: no two onsets half a cycle apart, so the cycle cannot be split into two equal halves at onsets. Chemillier and Truchet (2003) enumerate the words with it. Seven onsets in twelve pulses can never have it: there are six diameters and seven onsets (pigeonhole); so none of the ten bells has it, as Toussaint (2003) notes (asserted); he turns it into a measure, the count of equal bipartitions, fewer meaning more odd. Son clave has the property; E(11,24) has it; E(13,24) does not (all asserted). rhythm-systems.md records Toussaint's argument that off-beatness predicts preference among sub-Saharan timelines better than oddity does.

7 / THE MATHEMATICS

7.4 OFF-BEATNESS

On 12 pulses, the 2-, 3-, 4- and 6-gons (the possible even pulse streams) cover every position except 1, 5, 7 and 11. Toussaint (2003) counts onsets there as off-beatness; the standard pattern scores 3, the other nine at most 2 (asserted).

7.5 SWAP DISTANCE

A swap exchanges an adjacent onset and rest. Toussaint (2003) defines the distance between two rhythms with the same onset count as the fewest swaps between them, and builds phylogenies of the ten bells from it. Bembé to Tambú is one swap (asserted), as well as a rotation.

For a sequencer: SQRL-1's lane mutation ("one hit moves to an empty neighbour") is exactly one swap. A mutation limited to d swaps from the source keeps a variation recognisable, with a distance the player can set.

7.6 INTERLOCKING AS COMPLEMENT AND AS TILING

A hocket of two parts that never coincide and together fill the grid is a pattern and its complement (nyog cag, 4.1). Kotekan telu is a looser case: a cover with overlaps on the shared pitch. The general case, several rhythms whose translates tile the line, is the theory of rhythmic tiling canons (Vuza canons). D. T. Vuza's papers, Perspectives of New Music 1991-93, are unchecked here.

8 / FROM STUDY TO SAMPLER

8.1 WHAT RUDE TOOK

RUDE (plugins/rude, docs/rudiments.md, docs/gen_meters.py) ships, in its World family: tresillo E(3,8), cinquillo E(5,8), son and rumba clave 3-2, the Ewe bell (the standard pattern's rotation x.x.xx.x.x.x), Clapping Music, a kotekan illustration (r), konnakol khanda (5) and misra (7), a Noh hon-ji sketch (r), rachenitsa, daychovo and kopanitsa, maqsum, jhaptal, the diketo (r) and the Aka oddity pattern; and, from 0.3, the 35 Carnatic talas and six Turkish usuls. Its blends already implement two of this paper's ideas: Hocket (A's right hand interlocked with B's left, after kotekan) and Duet with Drift (phasing). Rotate, Mutate, Chop, and two strokes dividing one slot (as the grammar writes a diddle, the shape of taiko's do-ko), cover others.

8.2 WHAT SQRL-1 TOOK

SQRL-1's RUDIMENT (FM-1 firmware 1.1.0; fm1-flash/manual/note/rudiment.html) is a polymetric Euclidean machine: eight lanes, each with its own LEN, FILL (hits spread evenly) and ROT; DIV ratios per lane (x2, 3/2, 4/3, 3/4, 2/3, 1/2, 1/3, 1/4); sixteen preset RHYTHMs named after moons, every lane in them Euclidean; Grids' drum map (Gillet); logic lanes (AND, OR, XOR, NOR, A-B, FLIP, LATCH); PAD, WAKE and NAP; and a mutation that moves one hit to an empty neighbour. It has no named traditional pattern. Every Euclidean necklace in the CSV is reachable with LEN, FILL and ROT; the others need the step keys. A NOR lane reading a lane and an empty lane plays the complement, so nyog cag and the bell's silences are already reachable (derived from the manual's definitions, untried).

8.3 WHAT THEY COULD TAKE NEXT

The gaps this survey shows, in order of how much each adds per line of code:

- Rotation by onset (2.2). Both instruments rotate by steps.
- Stroke colour (3.5). Dum and tek over one onset set; RUDE has accents and the kick lane, SQRL has accents, neither has a per-step timbre choice on one lane.
- A locked reference lane (1.1, 2.1). Excluded from mutation, blends and EVOLVE.
- Per-step length (6.1). Ma, mitsuji's condensed beats, swing as a special case.
- Cycle hierarchy with the downbeat at the end (5.1). A colotomic preset family.
- Pitch contour per stroke (3.1). Needs a voice that can bend: a physical model, not GM.

CANDIDATES

Sequencer modes:

- Bell wheel: rotate a timeline by onsets; label rotations with their traditional names (the CSV has the ten bells and their wheel).
- Clave flip: swap 3-2 and 2-3 (half-cycle rotation) on one control.
- Reference lane: one lane exempt from mutation, blends and evolve.
- Darb mode: dum/tek/iss per step; fixed dum and tek, generated fills only in iss.
- Colotomy: kethuk, kenong, kempul, gong lanes from lancaran, ketawang or ladrang, gong on the last step, wela as a planned silence; irama as density-against-tempo.
- Khali / wela mute: drop a voice for a named section of the cycle.
- Ma: per-step length table plus a call event whose length places the next stroke.
- Kotekan splitter: split one line into polos and sangsih, by alternation (nyog cag) or by pitch with a shared middle (telu).
- Phase: rotate a copy one pulse every N bars (RUDE has it; SQRL could, per lane).

Generators:

- Oddity generator: 2s and 3s with no opposite pair (already a candidate in rhythms.csv).
- Aksak by class: pick prime, odd or even sum (Arom), then distribute 2s and 3s.
- Swap-distance mutation: variations at most d swaps from the source.
- Complement part: generate the second part as the first's complement; for the standard pattern this is a traditional timeline (E(5,12)).
- Off-beatness and evenness readouts: show both for the current lane; target either.
- Land on sam: a phrase of length p, three times with gaps, solved to end on beat one.
- Speech to rhythm: Bora-style pauses from a text's syllable structure; dùndún-style tone contours from tone-marked text.

Physical drum models:

- Dùndún: membrane with tension as a continuous control through the decay.
- Tabla dayan: loaded membrane with near-harmonic modes (Raman), stroke families by damping; bayan with heel pitch bend.
- Slit drum (manguaré): two-pitch wooden resonator.
- Iron bell / hoe blade: inharmonic bar and plate modes, very short.
- Tapan / davul: two heads, two beaters (thick stick, thin switch), one shell.
- Paired gongs and metallophones: two tunings that beat (ombak), gong hum for colotomy.
- Cuíca: stick-slip friction exciter on a membrane, pitch by finger pressure.
- Kotsuzumi: lacing squeezed during the stroke (needs a physical model).

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